Eur Phys J A (2004) **19**, s01, 77–79
Digital Object Identifier (DOI) 10.1140/epjad/s2004-03-013-1

EPJ A direct
electronic only

Generalized sum rules of the nucleon in the constituent quark model

M. Gorchtein[1], D. Drechsel[2], M. Giannini[1], and E. Santopinto[1]

[1] Universit a di Genova, Sezione INFN di Genova, via Dodecaneso 33, 16142 Genova (Italy)
[2] Institut für Kernphysik, Universität Mainz, Staudingerweg 45, 55099 Mainz (Germany)

Received: 16 Oct 2003 / Accepted: 14 Nov 2003 /
Published Online: 6 Feb 2004 –

Abstract. The sum rules serve a powerful tool to study the nucleon structure by providing a bridge between the statical properties of the nucleon (such as electrical charge, and magnetic moment) and the dynamical properties (e.g. the transition amplitudes to excited states) in a wide range of energy and momentum transfer Q^2. We study the generalized sum rules of the nucleon in the framework of the constituent quark model. We use two different CQM, the one with the hypercentral potential [1,2,3], and with the harmonic oscillator potential [4], both with only few parameters fixed to the baryonic spectrum. We confront our results to the model independent sum rules and to the predictions of the phenomenological MAID [5] model and find that in all the cases considered, in the intermediate Q^2 range (0.2-1.5 GeV^2), both CQM models provide a good description of the sum rules on the neutron.

PACS. 12.39.Jh Nonrelativistic quark model – 14.20.Gk Baryon resonances and helicity amplitudes

1 Introduction

In the recent years, precise measurements of single and double polarization observables for the photo- and electorabsorption have become possible. The inclusive cross section for the process ($ep \to eX$) can be written in terms of the four partial cross sections,

$$\frac{d\sigma}{d\Omega dE'} = \Gamma_V \Big[\sigma_T + \sigma_L - hP_x\sqrt{2\epsilon(1-\epsilon)}\sigma_{LT} - hP_z\sqrt{1-\epsilon^2}\sigma_{TT}\Big] \,, \qquad (1)$$

with $\sigma_T = \frac{\sigma_{1/2}+\sigma_{3/2}}{2}$, $\sigma_{TT} = \frac{\sigma_{1/2}-\sigma_{3/2}}{2}$, the virtual photon flux factor $\Gamma_V = \frac{\alpha_{em}}{2\pi^2}\frac{E'}{E}\frac{K}{Q^2}\frac{1}{1-\epsilon}$, and the photon polarization $\epsilon = \frac{1}{1+2(1+\nu^2/Q^2)\tan^2(\Theta/2)}$, where $E(E')$ denote the initial (final) electron energy, $\nu = E-E'$ the energy transfer to the target, Θ the electron c.m. scattering angle, and $Q^2 = 4EE'\sin^2(\Theta/2)$ the four momentum transfer. The virtual photon spectrum normalization factor is chosen to be $K = \frac{s-M^2}{2M}$, $h = \pm 1$ refers to the electron helicity, while P_z and P_x are the components of the target polarization. For a general and complete consideration of the nucleon sum rules we readress the reader to the review [6].

2 Constituent quark model

We study the generalized sum rules for the nucleon within a hyper central constituent quark model (HCCQM) previously reported in [1,2,3]. The model based on the lattice QCD inspired potential of the form $V(x) = -\frac{\alpha}{x} + \beta x + V_{hyp}$ and allows for a consistent description of the baryonic spectrum with a minimal number of parameters. Furthermore, to display the model dependence of a CQM calculation, we list also the results within the CQM with a harmonic oscillator (HO) potential. Within this model, we for the first time report a calculation of the longitudinal amplitudes and their contribution to the sum rules, details of which will be reported in an upcoming article.

The electromagnetic transition helicity amplitudes are defined as

$$A_{1/2} = -\frac{e}{\sqrt{2\omega}} < R, \frac{1}{2}|J_+|N, -\frac{1}{2} > \,,$$
$$A_{3/2} = -\frac{e}{\sqrt{2\omega}} < R, \frac{3}{2}|J_+|N, \frac{1}{2} > \,,$$
$$S_{1/2} = \frac{e}{\sqrt{2\omega}} < R, \frac{1}{2}|\rho|N, \frac{1}{2} > \,, \qquad (2)$$

where $\frac{1}{2}$, $\frac{3}{2}$ stands for the spin projection of the initial (nucleon) and final (resonance) hadronic state, and the definition was used, $J_+ \equiv \varepsilon^+ \cdot \mathbf{J} = -\frac{J_x+iJ_y}{\sqrt{2}}$.

We will present the results for the sum rules within the $zero-width$ approximation where one has for the contribution of a single resonance R to the partial cross sections

$$\sigma_L^R = 2\pi\delta(\nu-\nu_R)\frac{Q^2}{q_R^2}|S_{R,1/2}|^2$$

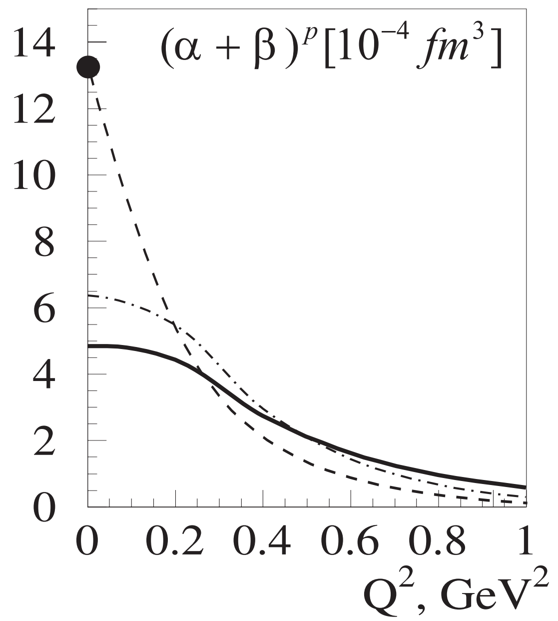

Fig. 1. Results for the sum of the proton polarizabilities $\alpha+\beta$ calculated in HCCQM (solid line), HO model (dashed-dotted line) in comparison with MAID (dashed line). The solid circle corresponds to the Baldin sum rule value at $Q^2=0$ [7]

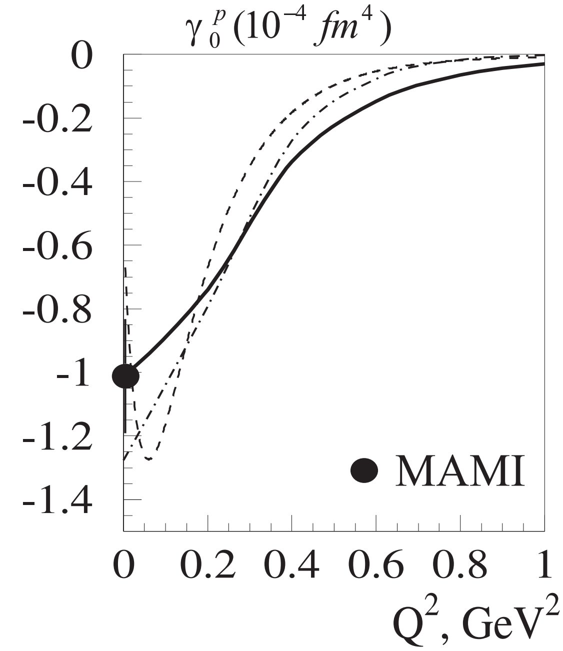

Fig. 2. Results for γ_0 on proton. Notation as in Fig. 1. The data point at $Q^2=0$ is from [7]

$$\sigma_{LT}^R = \sqrt{2}\pi\delta(\nu-\nu_R)\frac{Q}{q_R}\left(S_{R,1/2}\cdot A_{R,1/2}\right)$$
$$\sigma_{1/2,3/2}^R = 2\pi\delta(\nu-\nu_R)|A_{R,1/2,3/2}|^2\,, \tag{3}$$

with $\nu_R = \frac{M_R^2-M^2+Q^2}{2M}$, $q_R = \sqrt{\nu_R^2+Q^2}$, and M_R the resonance mass.

3 Sumrules for the forward polarizabilities of the proton

We start with the Baldin sum rule which relates the sum of the electromagnetic polarizabilities to the integral over the total photoabsorption cross section,

$$\alpha(Q^2)\;+\;\beta(Q^2)\;=\;\frac{1}{2\pi^2}\int_{\nu_0}^{\infty}\frac{K}{\nu}\frac{\sigma_T(\nu,Q^2)}{\nu^2}d\nu\,, \tag{4}$$

where $\nu_0 = m_\pi + \frac{m_\pi^2+Q^2}{2M}$ is the pion production threshold.

As it can be seen from Fig. 1, both constituent quark models fall short at $Q^2=0$ by a factor of 3, which is a consequence of lacking the large contribution of pion production. However, starting from $Q^2 = 0.2$ GeV^2 all three models give similar results.

We next turn to the sum rules with the helicity flip cross section σ_{TT}. In Fig. 2, we show the results for the forward spin polarizability,

$$\gamma_0(Q^2)\;=\;\frac{1}{2\pi^2}\int_{\nu_0}^{\infty}\frac{K}{\nu}\frac{\sigma_{TT}(\nu,Q^2)}{\nu^3}d\nu \tag{5}$$

In the case of γ_0, the small value phenomenologically comes about due to a strong cancellation of a large negative contribution of the $\Delta(1232)$ resonance, and a large positive contribution of near threshold pion production. Though the latter is not present in neither of the two presented quark model calculations, both do surprisingly well for this sum rule, as can be seen in Fig. 2, since almost all the transition helicity amplitudes in a CQM lack strenght, as compared to the phenomenological analysis. Therefore, the fact that the quark model results are consistent with the results of MAID in the shown range of Q^2 should be seen rather as a coincidence. It is interesting to note that, due to the characteristical for the HO potential gaussian form factors, the HO model closely reproduces the slope of the MAID curve.

4 Generalized GDH sum rule

The GDH sum rule relates the anomalous magnetic moment of the nucleon to the integral over its excitation spectrum,

$$-\,\frac{\kappa^2}{4}\;=\;\frac{M^2}{2\pi e^2}\int_{\nu_0}^{\infty}d\nu\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}\,, \tag{6}$$

thus providing a test of a quark model, since both left and right hand sides of this sum rule can be calculated. One of the possible generalizations of this integral to the case of finite Q^2 is

$$I_A(Q^2) = \frac{M^2}{\pi e^2}\int_{\nu_0}^{\infty}d\nu\frac{K}{\nu}\frac{\sigma_{TT}(\nu,Q^2)}{\nu}\,, \tag{7}$$

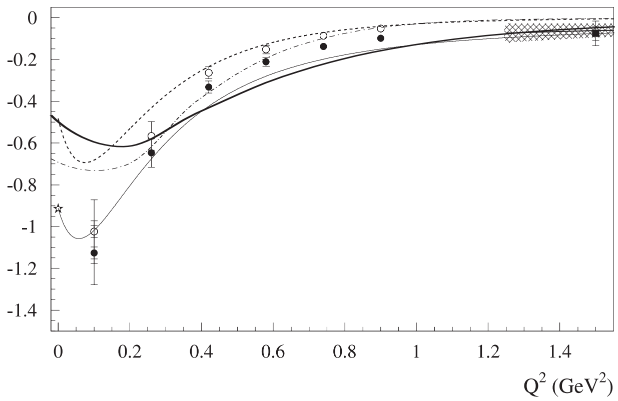

Fig. 3. Results for the generalized GDH integral I_A^n. Notation as in Fig. 1. The thin solid line corresponds to the experimental fit, as described in text. The solid line starting from 1.25 GeV^2 corresponds to the evaluation of the GDH integral using the data on the DIS structure functions (for the details, see [6]). The solid star represents the sum rule value at $Q^2 = 0$. The data points are from [8] (solid squares) and [9] (solid (full result) and open (resonant part with $W \leq 2$ GeV) circles)

In Fig. 3, we show the results for the GDH integral I_A on the neutron. As one can see, the sum rule at the real photon point is not obeyed in either quark model.

Starting from $Q^2 = 0.3$ GeV^2, the HCCQM practically follows the experimental fit, which assumes the following form (for details, see [6]):

$$I_A(Q^2) = I_A^{res}(Q^2) + 2M^2\Gamma_A^{as}\left[\frac{1}{Q^2+\mu^2} - \frac{c\mu^2}{(Q^2+\mu^2)^2}\right] ,$$
$$c = 1 + \frac{\mu^2}{2M^2\Gamma_A^{as}}\left[\frac{\kappa^2}{4} + I_A^{res}(0)\right] , \quad (8)$$

with $\mu = m_\rho$ and the resonance part as calculated with MAID. Due to the characteristical HO gaussian form factors having a more steep Q^2 dependence, the HO model is able to reproduce the data up to 1 GeV2, but falls short beyond this region.

5 Sum rule for the integral $I_3(Q^2)$

The only sum rule containing a prediction for the electric charge of the nucleon is

$$I_3(0) = \int_{\nu_0}^{\infty} d\nu \frac{K}{\nu}\frac{\sigma_{LT}}{Q} \underset{Q^2\to 0}{\to} \frac{e_N\kappa_N}{4} . \quad (9)$$

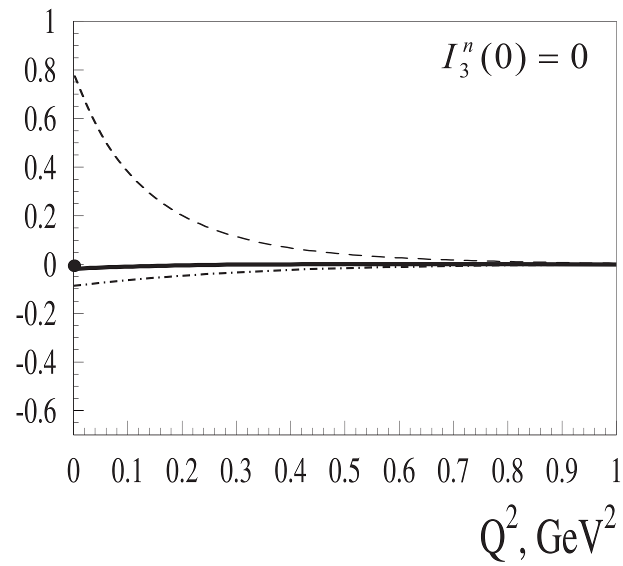

Fig. 4. Results for the I_3 integral for the neutron. Notation as in Fig. 1. Solid cicrcle corresponds to the sum rule value

In Fig. 4, we show the results for the I_3 integral on the neutron. While the MAID prediction is in complete disagreement with the sum rule value, one can see that the HCCQM result differs only slightly from zero, as required by the sum rule, and HO model gives a small negative value. Apart from the different potential of the two CQM models, the presented HO calculation does not take account of the hyperfine mixing of the wave functions which is responsible for the cancellation within this integral.

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